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Factorisation, parton entanglement and the Drell–Yan process

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Abstract. We discuss the angular distribution of the lepton pair in the Drell–Yan process, hadron+hadron → $\gamma^* + X \to l^+ + l^- + X$. This process gives information on the spin-density matrix $\rho^{(q,\bar{q})}$ of the annihilating quark–antiquark pair in $q + \bar{q} \to l^+ + l^-$. There is strong experimental evidence that even for unpolarised initial hadrons $\rho^{(q,\bar{q})}$ is non-trivial, and therefore the quark–antiquark system is polarised. We discuss the possibilities of a general $\rho^{(q,\bar{q})}$ – which could be entangled – and a factorising $\rho^{(q,\bar{q})}$. We argue that instantons may lead to a non-trivial $\rho^{(q,\bar{q})}$ of the type indicated by experiments.

1 Introduction

In this note we discuss the question of factorisation in the Drell–Yan process: [1]

$$h_1(p_1) + h_2(p_2) \to \gamma^*(k) + X \hookrightarrow l^+(q_+) + l^-(q_-)\,. \tag{1}$$

Here, h_1 and h_2 are the initial hadrons, γ^* is the virtual photon, l^+, l^- are the final state leptons ($l = e, \mu$) and X stands for the hadronic final state particles. The four-momenta are indicated in brackets. The basic underlying process is the annihilation of a quark–antiquark pair:

$$q(k_1) + \bar{q}(k_2) \to \gamma^*(k) \to l^+(q_+) + l^-(q_-)\,, \tag{2}$$

which is sketched in Fig. 1. Here we focus on the discussion of reaction (2) which is the lowest order process in the framework of the QCD improved parton model; see for instance [2]. For massless quarks we find that in (2) a lefthanded quark q_L can only annihilate with a righthanded antiquark $\bar{q}_R$ and vice versa.

The diagram of Fig. 1 is calculated by first evaluating the amplitude for (2) and folding it then with the parton distributions of the hadrons h_1, h_2. In early theoretical work, one usually assumed that for unpolarised hadrons h_1, h_2 the "parton beams" delivered by them are also unpolarised. We will call this the no-polarisation assumption. In the simplest approximation one furthermore assumes the partons q and $\bar{q}$ to be strictly collinear with the hadrons $h_{1,2}$. This leads to a well known angular distribution of the lepton pair in the rest frame of the virtual photon γ^*. With the polar and azimuthal angles of the outgoing l^+, θ and ϕ, one gets

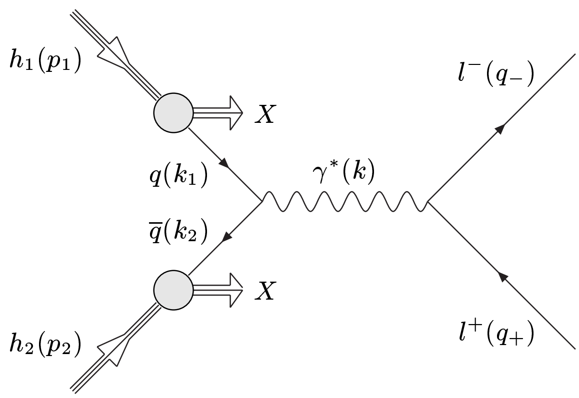

Fig. 1. The generic Drell–Yan process

$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{3}{16\pi}\left(1+\cos^2\theta\right). \tag{3}$$

Here and in (4) below we use the Collins–Soper reference frame [3] where the γ^* is at rest and the basis vectors $\boldsymbol{e}_{1,3}$ are defined by $\boldsymbol{e}_{1,3} = (\widehat{\boldsymbol{p}}_1 \pm \widehat{\boldsymbol{p}}_2)/|\widehat{\boldsymbol{p}}_1 \pm \widehat{\boldsymbol{p}}_2|$, with $\widehat{\boldsymbol{p}}_i = \boldsymbol{p}_i/|\boldsymbol{p}_i|$. The Collins–Soper frame is obtained from the h_1, h_2 CM system by a rotation free boost.

In general, the angular distribution of the l^+ is described by three functions λ, μ, ν which may depend on the kinematic variables of (1):

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$$\frac{1}{\sigma}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{3}{4\pi}\,\frac{1}{\lambda+3} \times \left(1+\lambda\,\cos^2\theta+\mu\,\sin 2\theta\,\cos\phi+\frac{\nu}{2}\sin^2\theta\,\cos 2\phi\right). \tag{4}$$

The LO pQCD result (3) implies for the functions $\lambda = 1$, $\mu = \nu = 0$. Higher order corrections in α_s change these values. But within the standard framework and using the no-polarisation ansatz one still finds [4] at NLO one relation among the coefficients in (4):

$$1-\lambda-2\nu = 0\,. \tag{5}$$

This Lam–Tung relation is almost unchanged at NNLO [5], and even holds for the inclusion of parton transverse-momentum and soft gluon effects [6]. However, the relation (5) is drastically violated in experiments [7–9].

In [10, 11] two at first sight quite different ideas have been proposed giving possible explanations for the violation of the Lam–Tung relation (5). It is the purpose of the present article to give a short review and a detailed comparison of these two approaches. We also sketch a calculation of instanton effects for the Drell–Yan reaction and pose some general questions concerning parton factorisation and entanglement.

2 Spin effects and factorisation

In [12] it was argued on general grounds that the assumption of unpolarised parton beams from a reaction with unpolarised initial hadrons is questionable due to possible vacuum effects. In particular, it was speculated that the fluctuating chromomagnetic vacuum fields which are due to the nonperturbative vacuum structure in QCD might lead to a correlated spin orientation of q and $\bar{q}$ in (2) before the annihilation. This would be in analogy to the Sokolov-Ternov effect [13], well known from e^+e^- storage rings.

In [10] this idea was expanded upon and confronted with experiments. A general two-particle spin-density matrix for the $q\bar{q}$ pair in (2) prior to the annihilation was assumed:

$$\rho^{(q,\bar{q})} = \frac{1}{4}\left\{\mathbb{1}\otimes\mathbb{1}+F_j\,(\boldsymbol{\sigma}\cdot\boldsymbol{e}_j^*)\otimes\mathbb{1} +G_j\,\mathbb{1}\otimes(\boldsymbol{\sigma}\cdot\boldsymbol{e}_j^*)+H_{ij}(\boldsymbol{\sigma}\cdot\boldsymbol{e}_i^*)\otimes(\boldsymbol{\sigma}\cdot\boldsymbol{e}_j^*)\right\}. \tag{6}$$

The quantities F_i, G_i and H_{ij} are real functions of the invariants of the problem. Here we work in the $q\bar{q}$ CM system and set

$$\boldsymbol{e}_3^* = \frac{\boldsymbol{k}_1^*}{|\boldsymbol{k}_1^*|}\,,\qquad \boldsymbol{e}_1^* = \frac{(\boldsymbol{p}_1^*+\boldsymbol{p}_2^*)\times\boldsymbol{e}_3^*}{|(\boldsymbol{p}_1^*+\boldsymbol{p}_2^*)\times\boldsymbol{e}_3^*|}\,,\qquad \boldsymbol{e}_2^* = \boldsymbol{e}_3^*\times\boldsymbol{e}_1^*. \tag{7}$$

Such a spin matrix will certainly affect the γ^*-production cross section from a $q\bar{q}$ state. The related production matrix in the $q\bar{q}$ CM system (the angular distribution of l^+ arises from the contraction with the lepton-production matrix) is

$$r_{ij}^{\gamma^*}(\boldsymbol{k}_1^*,\rho^{(q,\bar{q})};q\bar{q}) = \sum_{\text{colours, spins}}\frac{1}{9}\,\delta_{AA'}\delta_{BB'} \times\langle\gamma_i^*|\mathcal{T}|q(\boldsymbol{k}_1^*,\alpha,A)\,\bar{q}(-\boldsymbol{k}_1^*,\beta,B)\rangle\,\rho^{(q,\bar{q})}_{\alpha\beta\,\alpha'\beta'} \times\langle\gamma_j^*|\mathcal{T}|q(\boldsymbol{k}_1^*,\alpha',A')\,\bar{q}(-\boldsymbol{k}_1^*,\beta',B')\rangle^*\,. \tag{8}$$

Here $\alpha,\beta,\alpha',\beta'$ are the spin indices, A,B,A',B' the colour indices and we have assumed no polarisation in colour space. In LO the amplitude for γ^*-production reads

$$\begin{aligned}\langle\gamma_\mu^*|\mathcal{T}|q(k_1,\alpha,A)\,\bar{q}(k_2,\beta,B)\rangle &\equiv \langle 0|e\,J_\mu(0)|q(k_1,\alpha,A)\,\bar{q}(k_2,\beta,B)\rangle\\ &= e\,Q_q\,\delta_{AB}\,\bar{v}_\beta(k_2)\gamma_\mu u_\alpha(k_1)\,.\end{aligned} \tag{9}$$

Here $e\,J_\mu$ is the hadronic part of the electromagnetic current. The conventions for the Dirac spinors are as in [14] with $\alpha = \pm 1/2$ ($\beta = \pm 1/2$) representing the quark (antiquark) with spin orientation in the direction $\pm\boldsymbol{e}_3^*$.

With the standard no-polarisation assumption in spin space, one sets

$$\rho^{(q,\bar{q})}\big|_{\text{naive}} = \frac{1}{4}(\mathbb{1}\otimes\mathbb{1}) \equiv \frac{1}{4}(\delta_{\alpha\alpha'}\,\delta_{\beta\beta'})\,. \tag{10}$$

It was shown in [10] that a non-zero correlation coefficient,

$$\kappa \equiv \frac{H_{22}-H_{11}}{1+H_{33}}\,, \tag{11}$$

could easily explain the experimentally observed deviation from the Lam–Tung relation. Indeed, defining

$$\bar{\kappa} \equiv -\frac{1}{4}(1-\lambda-2\nu)\,, \tag{12}$$

one finds instead of (5) with (6) and (11)

$$\bar{\kappa} \approx \langle\kappa\rangle\,. \tag{13}$$

Here the average is over the parton longitudinal and transverse momenta [10]. In (13) we do not have an equality sign since higher order perturbative contributions give already a (very) small contribution to $\bar{\kappa}$ even for $\kappa = 0$.

A good fit to the data of [7, 8] could be obtained with the simple ansatz

$$\kappa = \kappa_0\,\frac{|\boldsymbol{k}_\mathrm{T}|^4}{|\boldsymbol{k}_\mathrm{T}|^4+m_\mathrm{T}^4}\,,\quad \kappa_0 = 0.17\,,\ m_\mathrm{T} = 1.5\,\mathrm{GeV}\,, \tag{14}$$

where $\boldsymbol{k}_\mathrm{T}$ is the γ^* transverse momentum in the hadronic CM system.

In [10] simplifying assumptions for the density matrix (6) were made:

$$F_2 = F_3 = G_2 = G_3 = H_{12} = H_{13} = H_{21} = H_{31} = 0\,. \tag{15}$$

This is irrelevant for the Drell–Yan process (2). One can easily show that the l^+ angular distribution is only sensitive to the two parameters H_{33} and κ.

Further discussions of possible QCD vacuum effects for the Drell–Yan and other reactions were given in [15,16].

The problem of the angular distribution in the Drell–Yan process was attacked from a different side in [11]. It was pointed out that there can be non-trivial spin and transverse-momentum correlations even inside an unpolarised hadron. In the notation of [11] the distribution of quarks (with lightcone momentum fraction x_1 and transverse momentum $\boldsymbol{k}_{1\mathrm{T}}$) inside an unpolarised hadron h_1 (with momentum p_1 and mass M_1) is given by a correlation function $\Phi(x_1, \boldsymbol{k}_{1\mathrm{T}})$, parametrised as follows:

$$\Phi(x_1, \boldsymbol{k}_{1\mathrm{T}}) = f_1(x_1, \boldsymbol{k}_{1\mathrm{T}}^2)\, \frac{\gamma^-}{2} + h_1^\perp(x_1, \boldsymbol{k}_{1\mathrm{T}}^2)\, \frac{\mathrm{i}\not{k}_{1\mathrm{T}}\, \gamma^-}{2M_1}\,, \tag{16}$$

where the transverse momentum is with respect to the plane spanned by the hadron momenta p_1 and p_2 in the hadronic centre of mass frame. In that frame p_1 and p_2 are predominantly in the lightlike n_+ and n_- directions, respectively. For details we refer to [11,17].

The function $h_1^\perp$ is the distribution of transversely polarised quarks with non-zero transverse momentum inside an unpolarised hadron. The subscript 1 on f_1 and $h_1^\perp$ indicates that these functions contribute at leading twist and should not be confused with the hadron label. The antiquark correlation function $\overline{\Phi}(x_2, \boldsymbol{k}_{2\mathrm{T}})$ is parametrised accordingly:

$$\overline{\Phi}(x_2, \boldsymbol{k}_{2\mathrm{T}}) = \bar{f}_1(x_2, \boldsymbol{k}_{2\mathrm{T}}^2)\, \frac{\gamma^+}{2} + \bar{h}_1^\perp(x_2, \boldsymbol{k}_{2\mathrm{T}}^2)\, \frac{\mathrm{i}\not{k}_{2\mathrm{T}}\, \gamma^+}{2M_2}\,. \tag{17}$$

The quark spin-density matrix $\rho^{(q)}$ can be obtained by projecting $\Phi(x_1, \boldsymbol{k}_{1\mathrm{T}})$ onto the basis $(\psi_{+\mathrm{R}}, \psi_{+\mathrm{L}})$, i.e. the right and left chirality components of the good field $\psi_+ = \frac{1}{2}\gamma^-\gamma^+\psi$ (see for instance [18,19]); and analogously for $\overline{\Phi}(x_2, \boldsymbol{k}_{2\mathrm{T}})$ and the antiquark spin-density matrix $\rho^{(\bar{q})}$. For given $\boldsymbol{k}_{1\mathrm{T}}$ and $\boldsymbol{k}_{2\mathrm{T}}$ one can boost to the frame (7), which leads (after appropriate normalisation) to

$$\begin{aligned} \rho^{(q)} &= \frac{1}{2}\left\{ \mathbb{1} + \frac{h_1^\perp}{f_1}\, \frac{x_1}{M_1}\, (\boldsymbol{e}_3^* \times \boldsymbol{p}_1^*) \cdot \boldsymbol{\sigma} \right\} \\ &\equiv \frac{1}{2}\left\{ \mathbb{1} + F_j\, (\boldsymbol{\sigma} \cdot \boldsymbol{e}_j^*) \right\}\,, \\ \rho^{(\bar{q})} &= \frac{1}{2}\left\{ \mathbb{1} - \frac{\bar{h}_1^\perp}{\bar{f}_1}\, \frac{x_2}{M_2}\, (\boldsymbol{e}_3^* \times \boldsymbol{p}_2^*) \cdot \boldsymbol{\sigma} \right\} \\ &\equiv \frac{1}{2}\left\{ \mathbb{1} + G_j\, (\boldsymbol{\sigma} \cdot \boldsymbol{e}_j^*) \right\}\,. \end{aligned} \tag{18}$$

For simplicity we have suppressed the arguments of the functions.

From (18) we arrive at $F_3 = 0 = G_3$ and for $i = 1,2$ at

$$\begin{aligned} F_1 &= -\frac{h_1^\perp}{f_1}\, \frac{x_1}{M_1} \boldsymbol{p}_1^* \cdot \boldsymbol{e}_2^*\,, \quad F_2 = +\frac{h_1^\perp}{f_1}\, \frac{x_1}{M_1} \boldsymbol{p}_1^* \cdot \boldsymbol{e}_1^*\,, \\ G_1 &= +\frac{\bar{h}_1^\perp}{\bar{f}_1}\, \frac{x_2}{M_2} \boldsymbol{p}_2^* \cdot \boldsymbol{e}_2^*\,, \quad G_2 = -\frac{\bar{h}_1^\perp}{\bar{f}_1}\, \frac{x_2}{M_2} \boldsymbol{p}_2^* \cdot \boldsymbol{e}_1^*\,. \end{aligned} \tag{19}$$

One observes that the function $h_1^\perp$ enters in the off-diagonal elements of $\rho^{(q)}$ and thus corresponds to RL and LR density-matrix elements.

In the approach followed in [11], the $q\bar{q}$ spin-density matrix is given by the tensor product of these two non-trivial one-particle spin-density matrices,

$$\rho^{(q,\bar{q})} = \rho^{(q)} \otimes \rho^{(\bar{q})}\,. \tag{20}$$

Clearly, non-zero $h_1^\perp$ implies that the standard no-polarisation ansatz (10) does not hold. Comparison of (20) and (18) with (6) shows that here $H_{ij} = F_i G_j$ for $i,j = 1,2,3$, and hence $H_{i3} = 0 = H_{3i}$ (due to $F_3 = 0 = G_3$).

The factorisation (20) of the spin-density matrix $\rho^{(q,\bar{q})}$ is usually implicitly assumed once factorisation of the dependences on hard and soft energy scales is demonstrated for a process. See for instance [20] for a discussion of factorisation of the spin-density matrix in the polarised Drell–Yan process (cf. in particular its (14)). For earlier discussions of issues concerning factorisation for processes where transverse momenta play a role see e.g. [21]. As said, for unpolarised hadrons it is standard to choose $F_i = 0 = G_i$. Using instead F_i and G_i of (19) in a tree level calculation of the Drell–Yan process leads to $\lambda = 1, \mu = 0$ and $\nu \neq 0$. The general expression for ν in terms of $h_1^\perp$ is given in [11], but here we will restrict to the case of Gaussian transverse-momentum dependence for illustration purposes. We assume that all transverse-momentum-dependent functions are of the form

$$f(x_i, \boldsymbol{k}_{i\mathrm{T}}^2) = f(x_i) \exp\left(-R^2 \boldsymbol{k}_{i\mathrm{T}}^2\right) \frac{R^2}{\pi}\,. \tag{21}$$

Moreover, we assume that the width of the Gaussian is the same for f_1 and $\bar{f}_1$ (the width will be called R_f^2) and similarly for $h_1^\perp$ and $\bar{h}_1^\perp$ (the width will be called R_h^2 and should be larger than R_f^2 in order to satisfy a positivity bound). This then leads to

$$\begin{aligned} \bar{\kappa} = \frac{\nu}{2} &= \frac{R_h^2}{4R_f^2}\, \frac{\boldsymbol{k}_\mathrm{T}^2}{M_1 M_2}\, \exp\left(-\left[R_h^2 - R_f^2\right] \frac{\boldsymbol{k}_\mathrm{T}^2}{2}\right) \\ &\quad \times \frac{\sum_a e_a^2\, h_1^{\perp a}(x_1) h_1^{\perp \bar{a}}(x_2)}{\sum_a e_a^2\, f_1^a(x_1) f_1^{\bar{a}}(x_2)}\,, \end{aligned} \tag{22}$$

where $\bar{\kappa} \equiv -(1-\lambda-2\nu)/4$ and $e_a = eQ_q$; see (12) and (9). We find again that the deviation from the Lam–Tung relation arises from an average κ (albeit in addition to higher order perturbative corrections). In (22) e_a denotes the charge of the quark with flavour a; the sum is over flavours and antiflavours (indicated by $\bar{a}$); and, we have used that $\bar{f}^a = f^{\bar{a}}$, i.e. the distribution of antiquarks of flavour $\bar{a}$ inside a hadron h is equal to the distribution of quarks of flavour a inside an antihadron $\bar{h}$.

Setting $R_f^2 = 1\,\mathrm{GeV}^{-2}$ and fitting the NA10 data [8] as done in [11], leads to a good fit for $R_h^2 - R_f^2 = 0.17 R_f^2$ and

$$\left\langle \frac{\sum_a e_a^2\, h_1^{\perp a}(x_1) h_1^{\perp \bar{a}}(x_2)}{\sum_a e_a^2\, f_1^a(x_1) f_1^{\bar{a}}(x_2)} \right\rangle = 0.02\,, \tag{23}$$

where we consider the average over x_1 and x_2. Assuming u-quark dominance and $h_1^\perp/f_1 \approx \bar{h}_1^\perp/\bar{f}_1$, this leads to the reasonable result that on average $h_1^\perp$ is approximately 14% of the size of f_1.

This result is of course dependent on the assumptions, but it serves the purpose of illustrating that the data can in principle be explained by a non-zero $h_1^\perp$. Hence, in order to experimentally discriminate between the two approaches of [10,11], more data are clearly needed, either from other kinematic regions or from other processes. In the next section we will elaborate on what is required and what are the opportunities for distinguishing between the two approaches.

3 Comparison of the two approaches

In this section we compare the approaches of [10,11]. Let us first of all emphasise that the ansatz of [11], given by (18) to (20) is perfectly compatible with the general ansatz (6) put forward in [10], but restricts $\rho^{(q,\bar{q})}$ to be factorising.

There is a further restriction in the ansatz (18) to (20). It requires $F_3 = G_3 = 0$. This comes about as follows. The correlation function Φ in (16) for the hadron h_1 is supposed to depend only on the momenta p_1, p_2, k_1 (actually only on the direction of p_2), the correlation function $\overline{\Phi}$ for h_2 only on p_2, p_1, k_2. From three four-vectors we can only form one axial vector in each case,

$$a^\mu_{1,2} = \epsilon^{\mu\nu\rho\sigma} p_{1\nu}\, p_{2\rho}\, k_{1,2\,\sigma}. \tag{24}$$

To form a pseudoscalar invariant we need all four independent four-vectors:

$$\mathcal{A} = \epsilon^{\mu\nu\rho\sigma} p_{1\mu}\, p_{2\nu}\, k_{1\rho}\, k_{2\sigma}\,. \tag{25}$$

Now F_3 and G_3 are in essence measuring the degree of longitudinal polarisation of the quark q and antiquark $\bar{q}$. Therefore, due to parity invariance of the strong interaction, F_3 and G_3 must be pseudoscalar quantities and thus linear in $\mathcal{A}$ (25). But in the ansatz (19) and (20) F_3 arises from the correlation function Φ of hadron h_1, see (16), and can thus only depend on three four-vectors, from which we cannot form a pseudoscalar invariant. Thus, with the ansatz (18) to (20), F_3 must be zero. The same holds for G_3. Therefore, the ansatz of [11] implies $F_3 = G_3 = 0$ and due to the factorisation of the $q\bar{q}$ matrix $H_{33} = 0$. In the general ansatz of [10] the density matrix can from the outset depend on all four four-vectors of the problem, there is a pseudoscalar invariant (25) available, and F_3, G_3 do not have to vanish. Obviously, also H_{33} does not need to be zero in the general approach.

As mentioned in Sect. 2, the Drell–Yan reaction (2) is only sensitive to the density-matrix element H_{33} and the combination κ (see (11)). Therefore, one way to check if the restricted form (20) of $\rho^{(q\bar{q})}$ is actually realised would be to measure H_{33}. But, as already mentioned in [10], the normalised angular distribution (4) of the lepton pair is practically only sensitive to κ. A factor of $1 + H_{33}$ enters in the cross section formula but influences mainly the absolute normalisation. This latter effect is difficult to measure due to uncertainties in the quark and antiquark distributions and in higher order contributions giving rise to the so-called K-factors. Thus we are left with one relevant parameter κ.

Different physical mechanisms were proposed in [10, 11] to produce a non-trivial $q\bar{q}$ density matrix with $\kappa \neq 0$. In [10] it was suggested that effects of the non-trivial QCD vacuum may be responsible for $\kappa \neq 0$. In [22, 23] model calculations using the general framework of [11] were performed showing that initial-state gluon exchange can produce $\kappa \neq 0$.

Let us see if on general grounds we can expect different behaviour for the observable quantity $\bar{\kappa}$ (12) from these two physical pictures. One possibility for comparison would be to study $\bar{\kappa}$ as a function of $\boldsymbol{k}_\mathrm{T}$. The ansatz given in (14) – taken literally – implies that $\bar{\kappa} \approx \kappa_0$ for large $\boldsymbol{k}_\mathrm{T}$. This is a very different behaviour than that expected from an underlying $h_1^\perp$ function, which is assumed to vanish for large quark transverse momentum, in accordance with the ansatz of factorisation of hard and soft energy scales in the process. This forces $\bar{\kappa}$ to vanish (at least at tree level) in the limit of large $|\boldsymbol{k}_\mathrm{T}|$. Higher order α_s corrections may modify this conclusion. However, as mentioned the NNLO corrections were shown to be small [5, 10]. Their (negative) contribution to $\bar{\kappa}$ was found to be well below 1% for $|\boldsymbol{k}_\mathrm{T}|$ values up to 3 GeV (see Fig. 6 of [10]). Therefore, one expects $\bar{\kappa}$ (possibly corrected for the small higher order perturbative contributions) to decrease. A constant $\bar{\kappa}$, that is both positive and large, for large $\boldsymbol{k}_\mathrm{T}$ would therefore be irreconcilable with the approach of [11]. In the general framework of [10] such a behaviour for $\bar{\kappa}$ would be possible but is certainly not required.

The dependence of $\bar{\kappa}$ on the other scale in the process, the lepton pair invariant mass (denoted by m_{γ^*} in [10] and by Q in [11]), may also be different in the two approaches. Unfortunately it is not clear what would be the generic Q^2 behaviour of $\bar{\kappa}$ due to vacuum effects. Regarding $\bar{\kappa}$ arising from non-zero $h_1^\perp$, the expectation is that it will decrease approximately as $1/Q$ for large Q. This is based on results from [24], where the influence of soft gluons on similar azimuthal spin asymmetries was considered. This means that although $\bar{\kappa}$ is not power suppressed at tree level, higher order α_s contributions effectively give rise to power suppression. Note that this is quite different from dynamical higher twist contributions, such as discussed in [25], which typically lead to $\nu \sim \mathcal{O}(\langle k_\mathrm{T}^2\rangle/Q^2)$ and therefore, are expected to be important only for Q values smaller than the experimentally measured range from 4 GeV up to 12 GeV. In any case, the fall-off or persistence of $\bar{\kappa}$ with increasing Q could be a discriminating feature, similar to the $\boldsymbol{k}_\mathrm{T}$ dependence.

A further possibility to differentiate between the two approaches is to investigate a possible flavour dependence of $\bar{\kappa}$ by varying the types of beams ($\pi^\pm$, p, $\bar{p}$). Clearly vacuum effects do not favour a flavour dependence. On the other hand, if the ratio $h_1^\perp/f_1$ varies for different flavours and different hadrons, then this could lead to an observable flavour dependence. Thus far only $\pi^- N$ data have been published, although [9] mentions also to have data for a π^+ beam at the same energy.

Both vacuum effects and non-zero $h_1^\perp$ could lead to a $\bar{\kappa}$ that varies as a function of (x_1, x_2). In addition, observable flavour dependence of this x_i dependence would arise if the ratio $h_1^\perp(x_i)/f_1(x_i)$ varies differently as a function of x for different flavours and different hadrons. This includes the possibility that $h_1^\perp(x_i)/f_1(x_i)$ changes its sign as a function of x_i, which would lead to sign changes in $\bar{\kappa}$ as a function of (x_1, x_2), even when restricting to only one particular process.

As a last point in this discussion we mention that since the approach of [11] is based on a factorised $q\bar{q}$ spin-density matrix (20), one can test this type of factorisation by measuring several related processes, such as semi-inclusive deep-inelastic lepton–nucleon scattering, where the $h_1^\perp$ function enters in combination with other functions [26, 27]. See also [28, 29]. In principle one can determine as many observables as unknown functions in order to extract $h_1^\perp$ and test the consistency of the factorised approach. Needless to say, this is quite a formidable task, but outlines of such a scheme have been discussed in some detail in [17].

Clearly it will also be very interesting to compare the predictions of the approaches of [10, 11] for other Drell–Yan type processes, for instance Z-production as well as γ^* + jet and Z+ jet production in hadron–hadron collisions.

4 Instanton model

Thus far we have explained in a rather general way, how a violation of the Lam–Tung relation (5) can arise, if the standard ansatz (10) does not hold. In both approaches [10, 11] the strength of the violation follows directly from a comparison with the experiment. It would be of great interest to calculate the relevant parameter describing the asymmetry (namely κ) in a certain model. For the approach of [11] this has been done in [22, 23] using a spectator model. It was shown that initial-state gluon exchange could give rise to a non-zero $h_1^\perp$ and a corresponding $\bar{\kappa}$ for the $p\bar{p}$ and $\pi^- p$ initiated Drell–Yan process. In this section we want to take a different approach, namely to outline a model calculation that is along the lines of [10].

We already discussed the possibility that in a non-trivial vacuum the spins of the partons might be correlated. An intriguing possibility to describe the vacuum structure is given by instantons. Instantons [30] are nonperturbative fluctuations of the gluon fields and well known to induce chirality-violating processes, absent in conventional perturbation theory [31]. Especially this feature of instanton-induced processes is a strong motivation to study the role of instantons as a source of spin correlations. Along similar lines, various remarkable effects induced by instantons were investigated. One can find for instance in [32] an estimate of certain single spin asymmetries and in [33] an estimate of the Pauli form factor of the quark. One also expects an impact of instantons on the question how the proton spin is built up from the spin and angular momenta of the constituents, see e.g. [34] or the review [35]. Recently, an estimate of an azimuthal spin asymmetry induced by instantons in semi-inclusive deep-inelastic scattering was presented in [36].

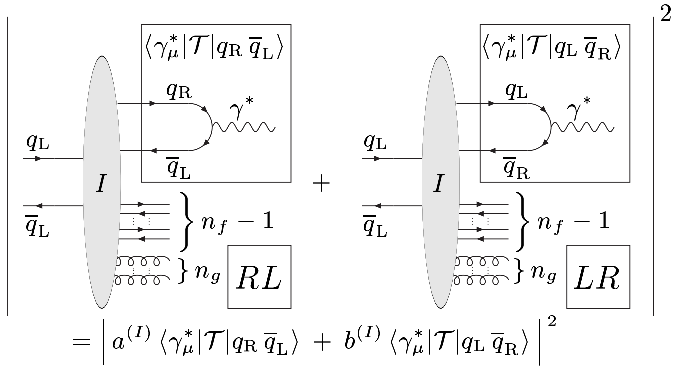

Fig. 2. The instanton-induced process $q_L + \bar{q}_L \to \gamma^* + (n_f - 1)\,[q_R + \bar{q}_R] + n_g\, g$

In a rather qualitative way, the Drell–Yan process was already investigated in an instanton background in [37]. There it was argued that even in the limit of high energy, instantons may lead to sizable effects, not suppressed by inverse powers of the energy. But it should be mentioned that spin effects do not play any specific role in [37].

Here, we want to emphasise that instantons might indeed violate the naive ansatz (10) via some additional terms to the spin matrix. The generic instanton-induced chirality-violating process which contributes in the Drell–Yan case reads for n_f active flavours (see (2) for the similar process in usual perturbation theory)

$$q_L + \bar{q}_L \to \gamma^* + (n_f - 1)\,[q_R + \bar{q}_R] + n_g\, g \qquad (26)$$

and is sketched in Fig. 2. The indices indicate the helicity of the quarks and antiquarks in the process. Of course, the process with R and L exchanged everywhere – induced by antiinstantons – must also be taken into account. The important point in our approach is not the significant complication of the final state in (26) which contributes to the final state X in (1), but the different helicity structure in the *initial* state.

We mentioned in Sect. 1 that neglecting quark masses in the process (2) only a quark and an antiquark with different helicities couple to the photon. So one can split the process (26) into two stages: during the first stage, the quark (or the antiquark) will change the helicity and afterwards the quark (antiquark) will interact in the usual way with the antiquark (quark). The final state will only change the size of the whole instanton contribution but not the structure of the related spin matrix $\rho^{(q,\bar{q})}$.

In Fig. 2 we show the two amplitudes contributing to the process (26) and for each amplitude the split into two stages. In the left part (labelled in (27) with (t)) the incoming *quark* changes the helicity. The right part (u) is of course similar but the incoming *antiquark* changes the helicity. It is sketched that both amplitudes factorise into an instanton part described by the coefficients $a^{(I)}$ and $b^{(I)}$ and chirality-conserving amplitudes, namely $\langle\gamma_\mu^*|\mathcal{T}|q_R\,\bar{q}_L\rangle$ or $\langle\gamma_\mu^*|\mathcal{T}|q_L\,\bar{q}_R\rangle$.

For the simplest instanton-induced process with $n_f = 1$ and $n_g = 0$ (see [38] for a detailed calculation of the related process in lepton–hadron scattering) one would expect a trivial connection between $a^{(I)}$ and $b^{(I)}$. For the general case this will change because of the more complex kinematics, related to the additional momenta of the final state partons.

The important point is that the two processes shown in Fig. 2 lead from the same initial to the same final states. Therefore these amplitudes must be added coherently. This gives in the cross section a term

$$\begin{aligned}
&\left(\mathcal{T}^{(t)}_{\mu\,\mathrm{LL}} + \mathcal{T}^{(u)}_{\mu\,\mathrm{LL}}\right)\left(\mathcal{T}^{(t)}_{\nu\,\mathrm{LL}} + \mathcal{T}^{(u)}_{\nu\,\mathrm{LL}}\right)^* \\
&= \left|a^{(I)}\right|^2 \langle\gamma^*_\mu|\mathcal{T}|q_\mathrm{R}\,\bar{q}_\mathrm{L}\rangle\langle\gamma^*_\nu|\mathcal{T}|q_\mathrm{R}\,\bar{q}_\mathrm{L}\rangle^* \\
&\quad + a^{(I)}\,b^{(I)\,*}\,\langle\gamma^*_\mu|\mathcal{T}|q_\mathrm{R}\,\bar{q}_\mathrm{L}\rangle\langle\gamma^*_\nu|\mathcal{T}|q_\mathrm{L}\,\bar{q}_\mathrm{R}\rangle^* \\
&\quad + a^{(I)\,*}\,b^{(I)}\,\langle\gamma^*_\mu|\mathcal{T}|q_\mathrm{L}\,\bar{q}_\mathrm{R}\rangle\langle\gamma^*_\nu|\mathcal{T}|q_\mathrm{R}\,\bar{q}_\mathrm{L}\rangle^* \\
&\quad + \left|b^{(I)}\right|^2 \langle\gamma^*_\mu|\mathcal{T}|q_\mathrm{L}\,\bar{q}_\mathrm{R}\rangle\langle\gamma^*_\nu|\mathcal{T}|q_\mathrm{L}\,\bar{q}_\mathrm{R}\rangle^* \,. \qquad (27)
\end{aligned}$$

Comparing the general amplitude (8) with the instanton-induced one (27), we get the following expressions for the density-matrix elements (the factor 1/4 arises from the averaging over the initial-state helicities):

$$\begin{aligned}
\rho^{(I)}_{\mathrm{RL\,RL}} &= \frac{\left|a^{(I)}\right|^2}{4}\,, \quad \rho^{(I)}_{\mathrm{LR\,LR}} = \frac{\left|b^{(I)}\right|^2}{4}\,, \\
\rho^{(I)}_{\mathrm{RL\,LR}} &= \frac{a^{(I)}\,b^{(I)\,*}}{4}\,, \quad \rho^{(I)}_{\mathrm{LR\,RL}} = \frac{a^{(I)\,*}\,b^{(I)}}{4}\,. \qquad (28)
\end{aligned}$$

For the calculation of the spin matrix, we have to add the contribution from the usual process without instantons in the background. The naive expectation related to (10) is $\rho_{\mathrm{LR\,LR}}|_{\mathrm{naive}} = \rho_{\mathrm{RL\,RL}}|_{\mathrm{naive}} = 1/4$ and $\rho_{\mathrm{RL\,LR}}|_{\mathrm{naive}} = \rho_{\mathrm{LR\,RL}}|_{\mathrm{naive}} = 0$. Adding this to the instanton-induced contribution we get finally

$$\kappa = -\frac{\rho_{\mathrm{RL\,LR}} + \rho_{\mathrm{LR\,RL}}}{\rho_{\mathrm{RL\,RL}} + \rho_{\mathrm{LR\,LR}}} = -\frac{2\,\mathrm{Re}\left(a^{(I)}b^{(I)\,*}\right)}{2 + |a^{(I)}|^2 + |b^{(I)}|^2}\,. \qquad (29)$$

An estimate of κ in the simplest case where $n_f = 1$ and $n_g = 0$ leads to $\kappa \neq 0$ and we expect the same to be true in general.

We want to mention that one can expect contributions to κ also from instanton-induced processes without any additional partons in the final state. In this case an instanton–antiinstanton pair is located only on one side of the cut appearing in the contributions to the cross section (in contrast to the squared amplitudes in Fig. 2 where one instanton will appear on each side of the cut). Hence, the quark and the antiquark will change the helicity on one side of the cut and we will also get off-diagonal contributions.

We summarise: The flipping of the helicity of one quark or antiquark in the initial state which occurs in the instanton-induced contribution to the Drell–Yan process should give rise to a non-zero matrix H_{ij} in (6) and finally to κ. As already mentioned, the Drell–Yan process (2) is not sensitive to F_i and G_i, hence we cannot say anything about instanton-induced contributions to F_i and G_i. A more careful analysis of the instanton-induced contributions to the Drell–Yan process including the complete final state in (26) is beyond the scope of the present paper. This and the question whether the instanton-induced processes lead to a factorising or entangled $q\bar{q}$ density matrix will be investigated elsewhere.

5 Summary

In this paper we have discussed the angular distribution of the lepton pair in the Drell–Yan process (1). We considered the lowest order reaction (2) and studied the influence of the quark–antiquark spin-density matrix on the lepton's angular distribution.

It is well known that a trivial spin-density matrix (10) is disfavoured by experiment [7–9]. Experiments are well described by $q\bar{q}$ spin-density matrices having the coefficient $\kappa \neq 0$. This can be achieved by a $q\bar{q}$ density matrix which is factorising into non-trivial q and $\bar{q}$ single-particle density matrices, as assumed in the ansatz of [11]. The ansatz of [10] is perfectly compatible with this, but would allow also for truly entangled $q\bar{q}$ pairs, that is a two-particle spin-density matrix which cannot be written as a tensor product of one-particle matrices. We have made a detailed comparison of the approaches [10,11] and have shown how they are related. We have discussed the underlying physical ideas and have outlined ways to check these ideas experimentally.

We have discussed instanton effects on the quark–antiquark density matrix and argued that these could induce spin correlations of the type indicated by experiments. The question whether instantons lead to a factorising or an entangled $q\bar{q}$ density matrix will be studied elsewhere.

We think that it is an important question to follow up how to determine from experiments the complete $q\bar{q}$ density matrix. For this other reactions besides the Drell–Yan process clearly are needed. We have given some discussion of this issue in Sect. 3. It would be fascinating if the $q\bar{q}$ density matrix turned out to be entangled. Thus, in this article we want to pose the question: can there be entanglement at the parton level in hadronic reactions?

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